# $\mathbf{\$ 1 8 9}$ or $\mathbf{\$ 3 9}$ plus $\mathbf{1 6 , 0 0 0}$ Frequent Flier Miles? Pricing in Combinations of Currencies to Lower Consumers' Perceived Cost 

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#### Abstract

The rising popularity of loyalty programs and related marketing promotions has resulted in the introduction of a number of new currencies (e.g., frequent flier miles, Hilton HHonors points, Diner's Club "Club Rewards") that people accumulate, budget, save and spend much like traditional paper money. As consumers are increasingly able to pay for goods and services such as airline travel, hotel stays and groceries in various combinations of currencies, understanding how shoppers respond to we call "combined-currency pricing" is becoming increasingly important to marketers.

This research is the first to explore how consumers evaluate transactions involving "combined-currency prices," or prices issued in multiple currencies (e.g., $\$ 39$ plus 16,000 miles). We present a formal mathematical proof outlining the conditions under which a price comprised of payments delivered in different currencies can be superior to a standard, single-currency price by either (a) lowering the psychological or perceived cost associated with a particular revenue objective (i.e., price), or (b) raising the amount of revenue collected given a particular perceived cost. Three studies, which include having actual airline travelers evaluate and make choices among prices issued in single and combined currencies, offer both experimental and empirical support.


KEY WORDS: Pricing, Combined-Currency Prices, Utility Theory, Loyalty Programs, Awards Programs, Incommensurate Resources, Mental Accounting, Frequent Flier Miles, Perceived Cost Function.
"The world has a new international currency: frequent flyer miles."
The Economist
May 2, 2002
"For millions of Americans, frequent-flier miles have become a second currency. In addition to piling them up by hopping on a plane, you can get them by making phone calls, buying toys, investing in mutual funds, taking out a mortgage, or renting cars."
"Meanwhile, don't be tempted by airline offers to sell you a ticket for a combination of miles and money. The deals are usually terrible."

Business Week
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Money has been around in one form or another since at least 9000 BC , and at one time or another cigarettes, cattle, stones, eggs, salt and porpoise teeth each has served as a negotiable instrument (Davies 1996). To be most useful, economists argue that a currency needs to be divisible, uniform and storable. Today, while almost all economies run on "fiat money," paper notes the government says are worth something, the immense popularity of marketing promotions and loyalty programs has resulted in the introduction of several new mediums of exchange that meet these criteria. From hotels (e.g., Marriott Rewards, Hilton HHonors) to credit cards (e.g., American Express Membership Rewards, Diner's Club's Club Rewards) and even sports teams (e.g., San Antonio Spurs Rewards), consumers are accumulating assets in a variety of novel currencies, which they budget, save and spend much like paper money.

Undoubtedly, the most ubiquitous alternative currency is frequent flier miles. One hundred million people around the globe, including one in three American adults, collect the nearly 500 billion miles distributed annually (WebFlyer.com 2002, Economist 2002). As of April 2002, the cumulative number of unredeemed miles was estimated worldwide at close to 8.5 trillion, which at current redemption rates - with no new miles being distributed - would take almost 23 years to clear (Economist 2002). In 2000 alone, 13 million free award tickets were issued (WebFlyer.com 2002). After comparing the relevant figures with all of the notes and coins in circulation around the globe,
frequent-flier miles could be considered the world's second biggest currency after the dollar, according to the Economist (May 4, 2002, p. 62). While loyalty programs are growing at a swift $11 \%$ per year, the fastest growing segment is that of "mileage consumers," not frequent fliers, as more than 18,500 U.S. businesses from telephone companies to car rental agencies distribute miles to their customers (Business Week 1999, WebFlyer.com 2002). These miles, given to recipients who never leave the ground, account for half of all of the miles earned (Economist 2002).

One direct consequence of the growing ubiquity of rewards programs is that consumers are increasingly able to pay for goods and services in a combination of currencies, not just dollars. For example, in the summer of 2002, Hilton HHonors offered members the opportunity to exchange either 600,000 points or 10,000 points plus $\$ 4,125$ for an 18 -carat gold, 1-carat solitaire diamond ring from the program's Diamond Collection (www.hilton.com/en/hhonors/rewards/diamond). Several more obscure currencies are progressively becoming interchangeable across programs, particularly in combination with frequent flier miles. A consumer can exchange American Express Rewards points (at a rate of 1-to-1.3) or Amtrak Guest Rewards (at 1-to-2) for HHonors points, which can subsequently be converted into frequent flier miles in affiliated programs, and spent in combination with dollars through a growing number of online malls and catalogs.

One such mall is Milepoint.com, an Internet exchange site backed by a group of prominent airlines that allows consumers to apply frequent flier miles as partial payment towards the purchase of more than 20 million products offered at participating merchants' sites. ${ }^{1}$ MileShopper ${ }^{\text {SM }}$ is an online catalog offering more than 300,000 brand name items from companies like Toshiba, Samsonite and Spalding for which shoppers can apply miles for up to $30 \%$ of the cost of their purchases. For example, in May 2001, the site offered $\$ 100$ off any Walt Disney World resort package of $\$ 199$ or more in exchange for 12,000 miles. Sites making similar offers at the time included MileSource.com and MyPoints.com among others.

Perhaps most familiar to frequent fliers are the deals offered by the airlines themselves. American Airlines AAdvantage, the first and largest frequent flier program in the world with more than 45 million members, is one of a number of programs routinely offering airline tickets for a combination of money and miles. For example, one NetSAAver fare advertised on the airline's web site during the winter of 2003 allowed fliers to purchase any ticket normally priced at $\$ 189$ for either (a) $\$ 189$, or (b) a combined-currency price of $\$ 39$ plus 16,000 miles.

Despite the growing popularity of prices issued in more than one currency, we know of no work that examines how consumers evaluate what we have labeled "combined-currency pricing" per se. In this research, we explore how consumers respond to prices offered in multiple currencies and determine the conditions under which a combined-currency price can be superior to a price charged in a single currency. We define superior as either (a) lowering the psychological cost to the customer associated with a particular revenue objective by the firm, or (b) raising the amount of revenue that can be collected given a particular psychological cost. Consider the following illustration. Imagine a consumer who is indifferent between spending $\$ 500$ or 25,000 miles on an airline ticket, but who would prefer paying $\$ 400$ plus 5,000 miles to either single-currency alternative. At $\$ 0.02$ per mile, the combined-currency price brings in the equivalent revenue to the airline, yet inflicts a smaller psychological cost to the consumer.

It is important to note that this consumer's preference for the combined-currency price indicates that each mile or dollar spent is not valued equally; the disutility of paying more dollars and/or miles increases as the payment in that currency increases. Accordingly, two essential requirements emerge for a combined-currency price to be superior: (1) the consumer does not value each unit within a currency equally, and (2) the perceived cost function for one of the currencies is convex for at least part of the range at issue. In the example above, it follows that the consumer is not spontaneously converting charges issued in one currency into increments of
the other currency at a constant rate, otherwise the flier would either: (a) be indifferent between the three choices, or (b) if his or her exchange rate was higher (lower) than the firm's, he or she should prefer to pay exclusively in dollars (miles). Consequently, if the firm understands the basic shape of the perceived cost functions for each currency (e.g., dollars and miles) within the applicable range of prices, using its own transfer function it can determine whether a price that combines currencies would be superior to a price issued in a single currency. This research proves this mathematically and demonstrates it empirically.

## CONCEPTUAL BACKGROUND

## The Value of Money and other Currencies

In classical economics, it is a truism that the utility of money is marginally decreasing, and therefore that utility is concave (Fennema and van Assen 1999). The principle of diminishing marginal utility (Stigler 1950) provides a rational argument for decreasing utility from equivalent incremental gains, and increasing disutility from equivalent incremental losses (a convex perceived cost function in our terms). However, since Prospect Theory (Kahneman and Tversky 1979) popularized the view that people evaluate changes in wealth relative to a reference point in much the same way the Weber-Fechner law of psychophysics says people respond to changes in physical stimuli such as light or sound, many psychologists have come to accept that the utility of losses is prone to diminishing marginal sensitivity (a concave cost function in our terms). Hence, while the concavity of utility for gains is rarely challenged, there is still widespread debate on whether utility is concave or convex for losses (e.g., expenditures).

Historically, utility functions have not been measured extensively (Farquhar 1984), but those attempting to do so have found empirical evidence supporting both concave and convex utility functions for losses (Davidson, Suppes and Siegel 1957, Green 1963, Swalm 1966,

Officer and Halter 1968, Fishburn and Kochenberger 1979, Tversky and Kahneman 1992, Abdellaoui 2000). After reviewing the evidence on both sides, Fennema and Van Assen (1999) concluded that, "Hence for losses one of the most basic aspects of utility, that is, whether marginal utility is increasing or decreasing, is as yet an unsettled question." While our work appears to support the economic prediction, the contradiction between diminishing marginal utility and Prospect Theory's conjecture of diminishing marginal sensitivity is well beyond the scope of this paper. While Prospect Theory pertains to changes in "wealth and welfare" utilizing money as its primary instrument, it does not address the issue directly of how people respond to multiple currencies or carriers of wealth simultaneously. Consequently, we prefer to view our findings as reinforcing the idea that utility is the result of a constructive process, which underscores how critical it is to understand the precise decision problem while studying the corresponding psychological mechanisms.

While we don't test it explicitly in this paper, we believe goal setting may be a common explanation for the convexity in the cost function necessary to make combined currency prices appealing. Heath, Larrick and Wu (1999) have shown that people are willing to exert more effort as they approach their goal and less effort as they move away from a goal. Just as someone whose goal is to do 40 sit-ups would be expected to exert more effort to do their $39^{\text {th }}$ sit-up than their $35^{\text {th }}, 4,000$ miles would mean more to someone with 20,000 miles than to someone with 10,000 miles, when the amount needed for a free ticket is 25,000 miles. Consider that a consumer who pays American Airline's NetSAAver fare of $\$ 39$ plus 16,000 miles rather than $\$ 189$ receives 0.94 cents, 0.23 cents per mile more than someone paying $\$ 39$ plus 7,000 miles rather than $\$ 89$, who receives only 0.71 cents per mile. This premium is akin to effort in the situp example as someone spending 16,000 miles is much closer to surrendering a free ticket ( 25,000 miles) than someone spending 7,000 miles. At the same time, American offered quantity
discounts to fliers looking to buy miles so members could "reach the awards [they] want - faster than ever before." Someone in need of 10,000 miles would pay significantly less per mile ( $\$ 250$ or 2.5 cents per mile) than someone needing only 1,000 miles ( $\$ 27.50$ or 2.75 cents per mile). These buy and sell rates are consistent with consumers ascribing a higher value to miles the closer they bring them to their goals.

Recognizing that the opportunities for spending miles or points are more limited than for dollars, we would expect salient goals or reference points (e.g., change in membership status, free flights and upgrades) to introduce convexities into the perceived cost function for these alternative currencies more readily than for money. This is not to say, however, that goals would never introduce convexities into the perceived cost function for money, as would be the case for someone saving up for a big screen television, for whom each dollar they accrue would mean progressively more as they neared their goal. Once the consumer surpassed the savings necessary to achieve their goal, one might expect the perceived cost function to become concave, with little incremental value to each additional dollar or mile accumulated (i.e., an S-shaped function).

While we mention goals as one plausible mechanism for introducing convexities into the perceived cost function, it is critical to clarify two important points regarding this research up front. First, our mathematical results depend only on the perceived cost function being convex for at least one of the currencies (it can be both) for at least part of the range at issue. Second, the underlying cause(s) for the convexity of a perceived cost function - whether due to goal setting, wealth effects, relativistic processing, mental accounting and budgeting, risk aversion or some combination of these and/or other explanations - has no impact on whether an optimal combined-currency price can exist or its derivation. In this research, we do not attempt to determine why the convexity occurs; what matters is only that some region of convexity in the
perceived cost function exists within the range of the expenditure for at least one of the currencies involved.

In practice, however, for combined-currency prices to be truly useful to the firm, two other conditions should be met. First, the firm should possess a transfer function or rate at which the firm values each mile surrendered that it uses to optimize globally over thousands of exchanges. This value has been reported to be $\$ 0.017$ (WebFlyer.com 2002), but is most often considered by consumers to be $\$ 0.02$, the standard selling price for most airlines and the amount at which Business Week (1999) advised fliers to value each of their miles. Given that the airlines redeem more than one billion miles every day, and the relatively small number of miles associated with any single promotion, we will assume a linear transfer function for ease of exposition, although a non-linear function can work as well. This is a reasonable assumption given the way most airlines manage their outstanding miles and in no way interferes with our goal of demonstrating how and why combined-currency prices can be superior from a consumer behavior standpoint. ${ }^{2}$

Second, consumers should not spontaneously convert charges issued in one currency into increments of the other currency, nor should they convert both simultaneously into some third currency. Work on "incommensurate resources" by Nunes and Park (2003) suggests consumers react to changes in alternative currencies much like they do for money, yet often do not spontaneously encode one currency in terms of another (e.g., frequent flier miles in dollar terms). Those authors found consumers made the conversion only when it was extremely easy to do so (e.g., a stable and salient exchange rate), or were extremely motivated to do so, as they might be for very large expenditures. Consequently, just as mental accounting and mental budgeting (Thaler 1985, Heath and Soll 1996) have demonstrated, consumers often behave as if their money were not perfectly fungible, and assets accounted for in different currencies are expected to have their own mental accounts and to be similarly non-interchangeable. Recent work by

Raghubir and Srivastava (2002) suggests that even when an exchange rate between monetary currencies is transparent and overtly provided, consumers are still susceptible to biases in their conversions. And while experience may attenuate a bias, it does not eliminate it.

Furthermore, differences in the aforementioned buy and sell rates, as well as among various combined-currency prices (see Table 1) support the assumptions that: (1) consumers don't always utilize the value of a mile to the seller or at some stable market rate when evaluating combined-currency prices, and (2) a consumer's subjective value for miles is likely to change depending on the quantity to be acquired or surrendered. Taken together, previous research and observed practices in the real world combine to suggest that the necessary conditions exist for opportunistic sellers to use combined-currency prices to shift the balance of payments among currencies (i.e., combined-currency pricing). They can do so in order to minimize the psychological pain associated with a purchase.

The rest of this paper is organized as follows. First, we present a formal mathematical proof outlining the conditions under which a combined-currency price is superior, and alternatively the conditions under which a price issued in one currency is superior. In this section, we outline how convexity in the perceived cost functions for one of the currencies involved opens the door for a superior combined-currency price: either by minimizing the psychological cost associated with a given revenue objective, or by maximizing the revenue collected given a particular psychological cost. The three studies that follow demonstrate that combined-currency prices can be preferred by consumers and support the predictions derived from our proofs.

More specifically, in Study 1, results from a laboratory study suggest that a combinedcurrency price can be preferred, particularly for prices involving relatively small amounts of dollars and miles. Conversely, single-currency prices are favored for relatively high prices. In

Study 2, we ask actual airline travelers to evaluate and make choices among prices issued in single and combined currencies. The results illustrate how combined-currency prices can indeed be superior, and how preferences shift systematically based on the magnitude of the price. Study 1 is limited in that respondents chose between a combined-currency price and only one singlecurrency alternative, and the results were aggregated across respondents. In Study 2, we offered actual fliers a complete range of choices, yet prices varied by individual and the results were aggregated across the range of prices. In Study 3, we replicate the principal results from Studies 1 and 2 utilizing a within-subjects design as well as an identical, complete menu of pricing options. In addition, we apply a more rigorous test of the assumption of convexity for at least one of the currencies involved. The paper concludes by pointing out some of the limitations of this research, offering some managerial implications and suggesting avenues for future research.

## COMBINING CURRENCIES TO LESSEN PERCEIVED COST

In this section, we show how non-linear valuations result in many instances where marketers - in order to secure a particular revenue objective with the least amount of psychological pain - should charge prices in a mixture of currencies (i.e., combined-currency pricing). We also describe situations in which a price charged in a single currency is optimal. We begin by examining the case in which the perceived cost function for both currencies is concave, followed by the case in which both cost functions are convex. We extend the discussion by describing the case in which one is concave while the other is convex, and conclude by describing the expected results when one currency's perceived cost function is S-shaped.

For simplicity and ease of exposition, we work with only two currencies: $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$. We assume that the perceived cost function for each increases monotonically (giving up more miles or dollars is worse, less of either is better) and that the company possesses some transfer function
(i.e., exchange rate) for the two currencies, which is linear (i.e., one unit of $\mathrm{C}_{1}$ is worth $\alpha$ units of $\mathrm{C}_{2}$ ). Without loss of generality, we can set $\alpha=1 . .^{3}$ The firm's target price or revenue objective can be described as a combination of $C_{1}$ and $C_{2}$ such that:

$$
\begin{equation*}
c_{1}+c_{2}=r \text {, where } c_{\mathrm{i}} \geq 0 \text { is the amount to be paid in currency } C_{\mathrm{i}} \text {. } \tag{1}
\end{equation*}
$$

From the consumer's perspective, based on our assumption that consumers do not convert the two into any meaningful common unit of measurement, the subjective value of $c_{1}$ and $c_{2}$ vary independently. Thus, the subjective loss or psychological cost $(E)$ associated with surrendering some combination of $c_{1}$ and $c_{2}$ can be written as:

$$
\begin{equation*}
E=f\left(c_{1}\right)+g\left(c_{2}\right) \tag{2}
\end{equation*}
$$

where $f$ and $g$ are strictly monotonically increasing continuous functions of $c_{1}$ and $c_{2}$, respectively, defined over the interval $[0, \infty)$ (i.e., $f^{\prime}>0$ and $g^{\prime}>0$ ). Further, $f(0)=g(0)=0$, and $f^{\prime}, g^{\prime}, f^{\prime}$, and $g^{\prime \prime}$ exist over their whole domain.

Let's assume the goal of the firm is to set a price that secures its revenue objective while minimizing the psychological cost to the consumer. The goal could just as easily be to maximize the revenue received given a fixed psychological cost, but practically speaking, we expect firms to begin with established revenue objectives, not perceived values, when developing combinedcurrency prices. Thus, the firm must solve the optimization problem:

$$
\begin{align*}
& \text { Min } E=f\left(c_{1}\right)+g\left(c_{2}\right) \\
& \text { st }: c_{1} \geq 0  \tag{3}\\
& \quad c_{2} \geq 0 \\
& c_{1}+c_{2}=r
\end{align*}
$$

A well-known mathematical result is that, for any given $r$, the solutions $\left(c_{1}^{*}, c_{2}^{*}\right)$ to equation (3) will be such that:

$$
f^{\prime}\left(c_{1}^{*}\right)=g^{\prime}\left(c_{2}^{*}\right) \text { for an interior solution, and }
$$

$$
f^{\prime}(0)>g^{\prime}(r) \text { or } g^{\prime}(0)>f^{\prime}(r) \text { for a corner solution (see Appendix A). }
$$

An interior solution will give rise to combining currencies, while a corner solution will give rise to a price assessed in only one currency. Therefore, the firm must determine when they are facing a corner solution and when they are facing an interior solution. This will depend on the subjective valuations of consumers (i.e., the shape of their perceived cost functions) for currencies $C_{1}$ and $C_{2}$.

In what follows, we show mathematically when a firm should charge a combinedcurrency price issued in some combination of currencies $C_{1}$ and $C_{2}$, rather than a price issued in one currency ( $C_{1}$ or $C_{2}$ alone) in order to extract the revenue objective, $r$, with the minimum psychological cost to the consumer. We will assume that $f(r) \leq g(r)$ purely for expositional purposes, as all claims can be transposed for those situations in which $g(r)<f(r)$. We proceed by first examining the case in which the perceived cost functions for both currencies are concave, before proceeding to the case in which both are convex. We then examine the case in which $f$ is strictly concave and $g$ is strictly convex over $[0, r]$, as well as the case in which $f$ is strictly concave and $g$ is S-shaped over $[0, r]$.

## CASE 1: Both $\boldsymbol{f}$ and $\boldsymbol{g}$ are strictly concave over $[0, r]$

Imagine the case of Wagner, a project engineer who travels frequently for work and has flown more than 100,000 miles in the past year. He is living comfortably on his nearly $\$ 100,000$ annual salary. Paying 5,000 miles or $\$ 50$ more or less for most things is not likely to mean much to him. In other words, whether he is buying an airline ticket using dollars or by redeeming miles, he is generally less sensitive to the marginal value attached to changes in both currencies (i.e., in this case both perceived cost functions are concave).

Proposition 1: When both $f$ and $g$ are strictly concave (i.e., $f^{\prime}>0, g^{\prime}>0$ and $f^{\prime \prime}<0, g^{\prime \prime}<0$ ) we have a corner solution. The solution is $(r, 0)$, the price should be assessed entirely in one currency $\left(C_{1}\right)$.

We proceed in two steps in order to prove Proposition 1. First, we show that when both $f$ and $g$ are strictly concave, there are no interior solutions. Second, we show that when $f(r) \leq g(r),(r, 0)$ is optimal.

Step 1 There are no interior solutions. If there were an interior solution $\left(c_{1}^{*}, c_{2}^{*}\right)$ then it would be the case that:

$$
\begin{aligned}
& f^{\prime}\left(c_{1}^{*}\right)=g^{\prime}\left(c_{2}^{*}\right) \\
& f^{\prime \prime}<0 \Rightarrow \forall \varepsilon>0, f^{\prime}\left(c_{1}^{*}-\varepsilon\right)>f^{\prime}\left(c_{1}^{*}\right) \\
& g^{\prime \prime}<0 \Rightarrow \forall \varepsilon>0, g^{\prime}\left(c_{2}^{*}+\varepsilon\right)<f^{\prime}\left(c_{1}^{*}\right) \\
& \Rightarrow f^{\prime}\left(c_{1}^{*}-\varepsilon\right)>g^{\prime}\left(c_{2}^{*}+\varepsilon\right) \\
& \Rightarrow f\left(c_{1}^{*}-\varepsilon\right)+g\left(c_{2}^{*}+\varepsilon\right)<f\left(c_{1}^{*}\right)+g\left(c_{2}^{*}\right)
\end{aligned}
$$

The last inequality contradicts the premise that $\left(c_{1}^{*}, c_{2}^{*}\right)$ minimizes $E$ since $\left(c_{1}^{*}-\varepsilon, c_{2}^{*}+\varepsilon\right)$ is a better solution and $c_{1}^{*}-\varepsilon+c_{2}^{*}+\varepsilon=c_{1}^{*}+c_{2}^{*}$. Hence, no interior solutions exist when both $f$ and $g$ are strictly concave (QED).

Step 2. If $f(r) \leq g(r)$ then $(r, 0)$ is optimal. If there are no interior solutions then either $(r, 0)$ or $(0, r)$ is optimal. It follows that $(r, 0)$ is optimal when $f(r) \leq$ $g(r)$.

Hence, when the psychological costs associated with each currency are strictly concave, the seller's decision is simple. For any given desired revenue, charge a price in one currency, the one that is valued least by the customer for the associated level of expenditure $r$. For any level of revenue, one currency will always weakly dominate the other. The best currency, however, may differ depending on the desired level of revenue, as the concavity of the cost functions needn't be the same. In Figure 1, currency $2\left(C_{2}\right)$ should be used for amounts smaller than 30, the point at which $f(r)=g(r)$, and currency $1\left(C_{1}\right)$ should apply for amounts larger than 30 . In other words, faced with a choice between paying $\$ 189, \$ 39$ plus 16,000 miles, or 20,160 miles, if Wagner
behaves optimally, we'd expect him to prefer to pay in either all dollars or all miles, depending on which currency he values less in that price range.

A corollary to this result is the following: for an individual to prefer a combined-currency price to a pure payment (one made in either currency alone), the psychological cost for the individual must be convex over at least part of $[0, r]$. Therefore, we now look at those cases in which the perceived cost function of either both, or only one currency is convex.

## CASE 2: Both $\boldsymbol{f}$ and $\boldsymbol{g}$ are strictly convex over $[0, r]$

Imagine Wagner plans to get engaged in a few months and begins diverting discretionary funds towards saving to buy his fiancée a sizable diamond engagement ring $(\$ 9,000)$. Simultaneously, he begins his quest to accumulate enough miles to upgrade the couple from coach to first class on his honeymoon trip to Brazil (100,000 miles). Suddenly, he is more sensitive to spending either an additional $\$ 50$ or 5,000 miles as relinquishing either might hinder or thwart his progress towards these goals. Wagner now attaches increasing marginal disutility to expenditures in both currencies. In this case, we describe how goals can create increasing marginal sensitivity, but it is easy to see how changes in income could have the same effect.

Proposition 2: When both $f$ and $g$ are strictly convex, a corner solution will exist only if $g^{\prime}(0) \geq f^{\prime}(r)$.

When $g^{\prime}(0) \geq f^{\prime}(r)$, we are in Situation 1, where a price issued in a single currency is optimal.

$$
\begin{array}{ll}
\text { Situation 1: } & \text { If } g^{\prime}(0) \geq f^{\prime}(r) \text { then }(r, 0) \text { is a corner solution } \\
\text { Proof: } & g^{\prime \prime}>0 \Rightarrow g^{\prime}(\varepsilon)>f^{\prime}(r), \forall \varepsilon>0 \\
& f^{\prime \prime}>0 \Rightarrow f^{\prime}(r-\varepsilon)<f^{\prime}(r), \forall \varepsilon>0 \\
& \Rightarrow g(\varepsilon)+f(r-\varepsilon)>f(r) \\
& (\text { QED })
\end{array}
$$

When $g^{\prime}(0)<f^{\prime}(r)$, we are in Situation 2, where a combined-currency price is optimal.
Situation 2: If $g^{\prime}(0)<f^{\prime}(r)$ there are no corner solutions

$$
\text { Proof: } \begin{aligned}
& \text { Since } g(r)>f(r) \text {, then }(0, r) \text { is not an optimal solution. } \\
& \text { Further, }(r, 0) \text { is not optimal either } \\
& \text { because: } \\
& \\
& g^{\prime}(0)<f^{\prime}(r) \Rightarrow \exists \varepsilon>0: g(\varepsilon)+f(r-\varepsilon)<g(0)+f(r) \\
& \\
& (\mathrm{QED})
\end{aligned}
$$

Hence, when $g^{\prime}(0)<f^{\prime}(r)$, the optimal solution will be to use both currencies in order to minimize the perceived or psychological cost (See Figure 2).

In this case, we cannot tell whether Wagner would prefer paying in a single currency, or to split his payments. He may want to preserve miles, as they are relatively harder to come by (he receives a regular pay check, but flies intermittently), and favor paying in all dollars whenever he can. Or, he may not quibble about small amounts of miles if he flies often, and instead favor combined prices that do not tax his frequent flier account. The important point is that, unlike the concave-concave case, even if he is behaving optimally, a combined price can be preferred.

## CASE 3: Either $\boldsymbol{f}$ or $\boldsymbol{g}$ is concave while the other is convex over $[0, r]$

Now let us imagine that Wagner has purchased the ring, yet is still several thousand miles short for the first class upgrades to Brazil. His fiancée's parents are paying for the wedding, so the disutility of each dollar he spends has diminished, while the psychological cost of each mile spent (taking him away from his goal) continues to increase as he nears the 100,000 -mile mark necessary for the upgrades for his honeymoon to Rio de Janeiro.

Let us examine the case in which $f$ is strictly concave and $g$ is strictly convex over $[0, r]$. When one of the two functions is concave $(f)$, while the other is convex $(g)$, there exist three possible optimal situations:

1. We have a corner solution $\left(0, r_{\mathrm{g}}\right) \forall r_{g}: g^{\prime}\left(r_{g}\right) \leq f^{\prime}(0)$
2. We have a corner solution $\left(r_{\mathrm{f}}, 0\right) \forall r_{f}: f^{\prime}\left(r_{f}\right) \leq g^{\prime}(0)$ and $f\left(r_{f}\right) \leq g\left(r_{f}\right)$
3. We have an interior solution in all other cases.

To more fully understand when each situation might apply, examine what happens to $E$ as $r$ increases. At 0 , no payment is made in either currency. If $f^{\prime}(0) \leq g^{\prime}(0)$, then all payments are always made in currency $C_{1}$ (situation 2) as $f(r)<g(r)$ and $\int_{0}^{\varepsilon} g^{\prime}(x) d x>\int_{r-\varepsilon}^{r} f^{\prime}(x) d x$ for $\forall \varepsilon<r$. This is a rather uninteresting case. In contrast, when $f^{\prime}(0)>g^{\prime}(0)$, we begin in situation 1 , where all payments are made in the second currency $\left(C_{2}\right)$. As $r$ increases, we move to situation 3, where payments are made in some combination of currencies $C_{1}$ and $C_{2}$, and finally progress into situation 2, where all payments are made in currency $C_{1}$.

Indeed, when $r$ moves away from 0 , the firm will be better off asking for payment in only $C_{2}$ as $g^{\prime}(x)<f^{\prime}(0), \forall x<r_{g}$. This will hold true until $g^{\prime}\left(r_{\mathrm{g}}\right)=f^{\prime}(0)$ (as $g^{\prime}>0$, this cannot hold indefinitely). As seen in Figure 3, after $r_{\mathrm{g}}$ the firm will minimize the psychological cost to the consumer by asking for payments made in a combination of both currencies $\left(C_{1}\right.$ and $\left.C_{2}\right)$ as $f^{\prime}(\varepsilon)<g^{\prime}(r-\varepsilon)$. As $r$ increases, the firm will reduce the amount to be paid in $C_{2}$ and increase the amount to be paid in $C_{1}$ until $r_{\mathrm{f}}$, the point at which the portion of the payments that are made in $C_{2}$ shrink to 0 . For any amount bigger than $r_{\mathrm{f}}$, only payment in $C_{1}$ should be requested.

In this case, Wagner may prefer to part with small amounts of miles (e.g., 1,000 ), but prefer paying cash for a shuttle ticket rather than give up 25,000 miles, as that would seriously impact his progress towards the free first-class upgrades. If he had accumulated the necessary miles, but not paid off the ring, we could expect the opposite, as he would prefer to use miles and apply the equivalent sum in cash towards the ring.

## CASE 4: Either $\boldsymbol{f}$ or $\boldsymbol{g}$ is S -shaped while the other is concave or convex over [0,r]

In Case 4, we examine two extensions: (1) the case where $f$ is strictly concave and $g$ is Sshaped over $[0, r]$ (see Figure 4), and (2) the case where $f$ is S-shaped and $g$ is strictly convex over $[0, r]$. When $f$ is concave and $g$ is S-shaped over $[0, r]$, the form of payment will depend on the derivatives of $f$ and $g$ at 0 .

Situation 1: If $f^{\prime}(0) \leq g^{\prime}(0)$, then we revert to the concave-concave situation (corner solutions where payments are made exclusively in $C_{1}$ or $C_{2}$ depending on whether $f(\mathrm{r})$ is smaller than $g(r)) .{ }^{4}$

Situation 2: If $g^{\prime}(0)<f^{\prime}(0)$ then we have a situation analogous to the concave-convex case:
a. We have a corner solution: $\left(0, r_{\mathrm{g}}\right) \quad \forall r: g^{\prime}\left(r_{g}\right) \leq f^{\prime}(0)$
b. We have a corner solution: $\left(r_{f}, 0\right)$

$$
\forall r_{f}: f\left(r_{f}\right)<g\left(r_{f}\right) \text { and } f^{\prime}\left(r_{f}\right)<g^{\prime}(0)
$$

c. We have an interior solution in all other cases.

When $f$ is S-shaped and $g$ is strictly convex over [ $0, r$ ], the solution will depend on the derivative of $g$ at its inflection point. If $g^{\prime}($ inflection $) \leq f^{\prime}(0)$, then the situation is analogous to the concaveconvex case where $f^{\prime}(0)<g^{\prime}(0)$. All payments should always be charged in currency $1\left(C_{1}\right)$.

$$
\text { If } g^{\prime}(\text { inflection })>f(0) \text { then we have three payment schedules: }
$$

1. split payment increasing in both $C_{1}$ and $C_{2}$ (à la convex-convex case) up to the point where $f^{\prime}\left(\mathrm{c}_{1 r}\right)=g^{\prime}\left(c_{2 r}\right)=g^{\prime}($ inflection $)$.
2. split payment increasing in $C_{1}$, decreasing in $C_{2}$ (à la concaveconvex case) up to the point where $f^{\prime}\left(r_{f}\right)=g^{\prime}(0)$.
3. payment in $C_{1}$ only (corner solution) for $r_{f}: f^{\prime}\left(r_{f}\right)<g^{\prime}(0)$.

We should also point out that when both $f$ and $g$ are S-shaped over $[0, r]$, we have a situation analogous to the S-shaped-convex case.

One of the primary goals of this research is to show that in numerous instances, the optimal price in terms of minimizing the perceived cost to the consumer entails setting a price that mixes $C_{1}$ and $C_{2}$, the two currencies involved. This could easily occur when the amount charged in $C_{1}$ falls in the concave region of its $S$-shaped perceived cost function and the charge in $C_{2}$ falls in the convex region. In Study 1, we illustrate how the subjective value that consumers ascribe to differing amounts of an individual currency (i.e., money, miles) appears not to be linear, resulting in a situation where both interior and corner solutions can exist at different points over the range of prices for the same two currencies.

## STUDY 1

As described earlier, given two currencies, if the consumer's perceived cost function includes a convex region for one currency, the firm can impose the minimum psychological cost associated with a particular revenue objective through the use of a combined-currency price. With this in mind, Study 1 is designed to illustrate two key points. First, for dollars and frequent flier miles, we show that an interior solution can exist such that consumers prefer a combinedcurrency price. We expect to see signs of increasing marginal disutility for miles and/or money when this type of pricing structure is preferred. Second, we show that a corner solution can also exist, where a price in one currency or the other dominates.

## Method

Subjects. Participants in this study were 670 undergraduate business students enrolled in an introductory marketing course at a major West Coast university. Of the 670 students, 363 or $58 \%$ of participants reported possessing an active frequent flier account containing miles. We limited our analysis to these participants' responses in an attempt to insure
that our results would only reflect choices from airline loyalty program members with experience collecting miles. ${ }^{5}$

Stimuli and design. Participants completed a scenario-based, paper and pencil study in which they were asked to imagine they were purchasing an airline ticket with either dollars or frequent flier miles. At the onset of the experiment, each respondent was asked whether he or she had a frequent flier account, and if so to indicate how many miles they had in their largest account. This would allow us to control for experience with the alternative currency.

Participants were subsequently instructed to disregard their personal accounts and to assume they possessed enough miles and money to accommodate whichever pricing option they preferred. The cost of the ticket was either low ( $\$ 250$ or 25,000 miles) or high ( $\$ 1,000$ or 100,000 miles). The base price did not include a mandatory surcharge, which could be paid in either dollars $(\$ 50)$ or miles $(5,000)$. A pilot test found respondents from the same sample population valued 5,000 at approximately $\$ 50$, or $\$ 0.01$ per mile, which was consistent with our expectations. Respondents' task was to choose between pricing schedules, which included the surcharge in either dollars or miles. The scenario with miles as the base cost read as follows:

You are on the phone with an airline securing a round-trip ticket across country to attend the funeral of an uncle you really liked and admired. While you can't leave town for two days, you must pay for the ticket today. The price of the ticket is $25,000[100,000]$ miles. The agent on the phone tells you that in order to have your ticket request expedited, which would be necessary to receive your ticket in time for your departure, you will need to surrender either an additional 5,000 miles or pay $\$ 50$. How would you prefer to pay?

$$
25,000+\$ 50 \_\quad 25,000+5,000 \text { miles }
$$

An additional choice scenario required respondents to choose between paying entirely in dollars or entirely in miles. The basic scenario read as follows:

You are on the phone with an airline securing a round-trip ticket across country to attend the funeral of an uncle you really liked and admired. While you can't leave town for two days, you must pay for the ticket today. There are two possible price combinations with which you can pay for your ticket. First, you can pay $\$ 250$ [ $\$ 1,000]$ for the ticket plus an extra $\$ 50$ to have your ticket order expedited, which would be necessary to receive your ticket in time for your departure. Or you can surrender 25,000 [100,000] miles for the ticket, and an additional 5,000 miles to have your ticket expedited. How would you prefer to pay?

The remaining two choice combinations were included for completeness, which forced respondents to choose between paying either $\$ 250+5,000$ miles or 25,000 miles $+\$ 50$, and $\$ 1,000+5,000$ miles or 100,000 miles $+\$ 50$. A number of the participants received more than one of the choices, which were rotated and counterbalanced.

If respondents possessed a linear transformation function between money and miles, and the exchange rate were constant at one mile equaling one cent, they should have always be indifferent between the two pricing schedules. If the perceived cost function were linear, and on average respondents valued each mile at more (less) than one cent, they should always prefer to pay the price that includes more (less) money. Our prediction, however, was that for relatively small revenue objectives or prices ( $\$ 250$ or 25,000 miles), respondents would prefer a combination of miles and dollars to payments exclusively in one currency, requiring convexity for one of the currencies in this range. This implies an interior solution. On the other hand, when the revenue was relatively large ( $\$ 1,000$ or 100,000 miles), we expected respondents to prefer paying in one currency alone, which would imply concavity in this region. This implies a corner solution. Note that this pattern of results would be consistent with an S-shape perceived cost function for miles or money, or both.

## Results

The results are summarized in Table 2. As predicted for Part 1 (the first column), for payments in the Relatively Low Total Cost conditions, a significant majority of respondents preferred the combined-currency prices to charges issued in a single currency. This suggests that for expenditures involving only 5,000 miles, respondents preferred paying in miles, while for expenditures from 25,000 to 30,000 miles, respondents preferred paying the surcharge in dollars. This result suggests the marginal value of miles increases at an increasing rate (convexity).

Conversely, in the Relatively High Total Cost conditions, respondents preferred paying in a single currency (i.e., $\$ 1,050$ and 105,000 miles respectively), implying the incremental 5,000 miles is worth more alone than when added to 100,000 miles, or the value of miles increases at a decreasing rate (concavity). These results conform to the predictions from our mathematical proofs, and are consistent with an S-shaped perceived cost function for miles. If the perceived cost function is convex over small amounts of miles, and the perceived cost function for money is assumed to be concave, we would expect an interior solution (respondents preferring to pay in bundles of money and miles) rather than a corner solution (a preference for a price in either solely money or miles). Indeed a combined-currency price is preferred in this range. And, given that the perceived cost function appears concave for large amount of miles, we would expect the opposite to occur in the $\$ 1,000$ and $100,000-$ mile conditions. In this range, most respondents preferred prices in one currency alone (a corner solution). We should point out that this pattern of results is also consistent with two S-shaped cost functions or two convex cost functions, where the value of money increases faster than that of miles.

Notice that, when limited to single-currency prices, for relatively small amounts (\$300, 30,000 miles) people preferred paying entirely in dollars. Conversely, for relatively high prices ( $\$ 1,050,105,000$ miles), people preferred to pay using miles exclusively (See Table 2, Part 2).

This implies their valuation for dollars did increase faster than for miles as the amount to be spent increased. Not surprisingly then, when forced to choose between combined-currency prices (See Table 2, Part 3), for relatively small amounts, subjects favored the payment comprised principally of dollars ( $75 \%$ versus $25 \%$ ). The reverse was true when the combined-currency prices were relatively high ( $\$ 1,000$ plus 5,000 miles versus 100,000 miles plus $\$ 50$ ) with far more respondents preferring the pricing schedule comprised principally of miles ( $85 \%$ versus $15 \%$ ). While the exact turning point where people switched from preferring to pay in dollars to paying in miles may depend on the exchange rate we utilized (\$0.01), we expect the general result to hold; the exchange rate between the two currencies among consumers is not constant.

We should remind the reader that relativistic processing (Nunes and Park 2003) may have affected how respondents evaluated the surcharge ( $\$ 50$ in addition to $\$ 250$ versus $\$ 1,000$ and 5,000 miles in addition to 25,000 versus 100,000 miles). The changing nature of these valuations is entirely consistent with our hypothesizing. Recall that in our pretest, subjects valued 5,000 miles and $\$ 50$ as equivalent. Yet when added to 25,000 , they would prefer to surrender the $\$ 50$ and retain the 5,000 miles. This suggests increasing marginal disutility and predictably, a combined-currency price is preferred. When added to 100,000 miles respondents preferred to pay the 5,000 miles, or a single-currency price, which is consistent with diminishing marginal disutility. The same pattern holds true for a cash surcharge.

To rule out the possibility that subjects' current frequent flyer mile holdings influenced our results, we ran a separate ANOVA for each choice, where the average number of miles held by those favoring one option was compared to the average number held by those favoring the opposing option. None of these comparisons was significant ( $p$-values ranging from 0.12 to 0.96 ; average $p$-value $=0.58$ ), suggesting that their decision did not depend on the number of miles in their accounts. In addition, we ran eight separate logit models on choice using the individual's
holdings in miles as a covariate to determine whether holdings in miles could predict choice. None of the coefficients was significant ( $p$-values ranging from 0.15 to 0.96 ; average $p$-value $=$ $0.58)$.

## Discussion

The results from Study 1 illustrate how an interior solution can exist, resulting in the case where a combined-currency price is preferred. While we cannot say with certainty whether the perceived cost function is convex for miles, money or both, mathematically we have demonstrated that convexity must exist for one or both currencies in the low-cost $(\$ 300 / 30,000-$ mile) range. This convexity may continue into the relatively high-cost range (\$1,050/105,000mile), although the preference for single-currency prices among most subjects is also consistent with both functions being concave in this range, suggesting an S-shaped perceived cost function for one of the currencies, which we suspect is miles.

## STUDY 2

In Study 1, respondents were undergraduates, albeit business students, whose relative inexperience with transactions involving miles may have affected their decision-making. Respondents chose among price schedules similar to what airlines offer (e.g., $\$ 189$ or $\$ 39$ plus 16,000 miles) and not a full range of options, which would include both single-currency prices as well as the combined-currency price. In Study 2, we extend the results of Study 1 by: (1) surveying members of a particularly relevant test population; (2) allowing respondents to choose between paying the combined-currency price or paying in either of the two currencies involved (money or miles); and (3) examining how their preferences shift as the revenue objective
represented by these pricing options changed. The revenue objective or price was determined by the actual price of the ticket held by flier surveyed.

Given our findings from Study 1, we hypothesized that people's preferences for combined-currency prices would diminish as the revenue objective increased, and that those favoring a single-currency price would prefer paying in dollars for relatively low-priced ticked and yet in miles for relatively high-priced tickets. Recall that in Study 1, when limited to singlecurrency prices, people preferred paying entirely in dollars for small amounts, but entirely in miles for large amounts, suggesting their valuation for miles diminished faster than for dollars as the amount to be spent increased.

## Method

Subjects.
This experiment was run in a real world setting in an attempt to insure external validity. Participants were 164 passengers on commercial flights that were scheduled to depart from a major West Coast airport (this was prior to 9-11). The passengers were approached at the airport prior to their departure and asked to participate voluntarily in an academic study of airline ticket purchasing behavior. Participants were screened on the basis of whether or not they maintained a frequent flier account in which they accrued miles - only those who possessed frequent flier accounts participated. Again, we expected a significant proportion of fliers to favor a combined-currency price.

Stimuli and design. At the onset of the experiment, respondents were asked to recall what they had paid for the ticket that brought them to the airport at the time of the survey (all participants were waiting to take a flight when asked to participate). After they stated the price paid in either dollars or miles (only two flyers redeemed miles), the experimenter, unbeknownst
to the respondent, converted this amount into an equivalent price in the opposing currency at a rate of $\$ 0.02$ per mile (e.g., $\$ 200$ and 10,000 miles), as well as a combined price with 50 percent paid in each currency (e.g., $\$ 100$ and 5,000 miles). For simplicity, we limited their choices to dollars only, $50 \%$ dollars $/ 50 \%$ miles, or miles only. The participant was then asked which of three prices they would have preferred to pay if they had been offered the choice (e.g., $\$ 200$, $\$ 100$ plus 5,000 miles, or 10,000 miles).

## Results

Overall, we find that $24 \%$ of respondents preferred to pay in dollars only, $34 \%$ preferred to pay in miles only, and $42 \%$, or the largest segment of travelers surveyed, preferred paying in a combination of currencies (see Figure 5a). A primary goal of Study 2 was to explore how and when people's preferences shift among pricing options (dollars only, a combined-currency price, or miles only) across the various amounts to be paid. Therefore, we examine the probability that a person surveyed would choose a particular pricing option as a function of the price paid. The data were analyzed using the categorical modeling procedure of the SAS statistical software package (CATMOD), which allowed us to fit a multinomial logit model using choice, or the probability of choosing a particular pricing schedule, as the dependant measure and price as independent variable. The choice of one payment option over another varied significantly with price $\left(\chi^{2}=18.23, \mathrm{p}<0.001\right)$.

In order to more easily interpret the results we computed the choice probability for tickets ranging from $\$ 0$ to $\$ 2,500$, the highest price reported among those surveyed and graphed them in Figure 5b. As is clear from the figure, for small dollar amounts (Price $<$ approximately \$300), straight dollar payments were the most likely option. For intermediate ticket values ( $\$ 300<$ Price $<\$ 1,200$ ), mixed payments were preferred. Finally, for relatively more expensive tickets (Price $>$
$\$ 1,200$ ), consumers preferred to purchase the tickets using miles alone. Hence, we find support for our hypothesis, which is consistent with the results from Study 1 and our general framework.

We should note two caveats regarding the interpretation and generalization of these results. First, we used an exchange rate of two cents per mile. This will affect the relative attractiveness of paying in miles rather than dollars. A lower value, such as one cent per mile, would be likely to shift the curves depicted in Figure 5b to the right, such that strict dollars payments are preferred over a larger domain, and strict miles payment are preferred over a smaller domain. We would also expect the net impact on mixed payments to be a shift to the right, but it is not possible to tell whether they would be preferred over a wider or narrower range of ticket prices without collecting more data. Second, the only form of combined-currency prices we investigated were equally balanced, or a $50 / 50$ split. It is possible that other mixtures (e.g., $80 / 20$ ) would be preferred to any of the three forms tested (we investigate this further in Study 3). Hence, it is likely that the range of prices over which a combined-currency price is preferred is wider than highlighted by our experiment.

## Discussion

First and foremost, the results from Study 2 reinforce the notion that a combined-currency price can be the preferred option among consumers. The results also suggest preferences among competing price schedules can depend on the amount consumers intend to spend. Just as in Study 1 , surrendering primarily dollars is preferred at the low end, while paying primarily in miles is preferred at the high end. In the middle range, combined-currency prices are most widely favored. In both Studies 1 and 2, however, the results emerge from data aggregated across decision makers. Therefore, we cannot say with certainty that the same patterns would hold at the individual level.

## STUDY 3

Study 3 addresses many of the limitations of Studies 1 and 2 by utilizing a within-subject design in which respondents, who were all frequent fliers with sizable mile accounts, rank ordered the exact same set of price schedules, including both single-currency price options and several different combined-currency options. While we do not explore how the revenue object affects choice in this study, we do apply a more rigorous test of the assumption of convexity for each consumer's set of preferences.

## Method

Subjects. Participants in this study were 113 full-time MBA students at a major West Coast university. Of those surveyed, 14 did not possess frequent flier accounts and were excluded from the analysis. The average number of programs in which the remaining 99 respondents were enrolled was 2.5 (median $=3$ ), having collected miles for an average of 7.5 years (median $=6$ ). At the time of the survey, they possessed an average of 58,000 miles (median $=35,000)$ in their largest frequent flier account. Every one of the 99 members had at one time or another flown using a ticket secured with miles. Consequently, our analysis includes only experienced fliers familiar with dealing in transactions involving frequent flier miles. All respondents participated voluntarily.

Stimuli and design. Respondents were first-year MBA students who were scheduled to participate in an overseas program at the conclusion of the spring 2002 semester in which they would visit one or more countries in the Asia-Pacific region. Each had already purchased their travel package through the university. Their package price did not separate out the charge for airfare (i.e., they did not know what they paid for their airfare), but the survey asked students to
imagine that in the future, successive student groups (e.g., next year's class) would be offered the opportunity to buy their ticket separately.

Additionally, they were told that future groups might possibly be able to choose from various prices comprised of payments made in frequent flier miles and/or dollars, and that the program office would like to gauge their preferences as an indicator for how to handle travel arrangements in the future (i.e., which price schedules to offer). They were instructed to assume the flier would own enough miles to cover any option and that future participants would be able to travel on their preferred airline, no matter which option they favored. Respondents then ranked five price schedules in terms of their preferences ( 1 for most preferred price to 5 for least preferred price). The five price schedules presented were: $\$ 700, \$ 560$ plus 7,000 miles, $\$ 350$ plus 17,500 miles, $\$ 140$ plus 28,000 miles, or 35,000 miles. A price of $\$ 700$ was entirely in line with what was actually paid by the school for most tickets.

## Results

The primary purpose of study 3 was to do a within-subject test of our hypotheses. Given a series of choices which are equivalent to the firm in their value, if consumers have strictly concave valuations for miles and money, they will only pick as their first choice one of the pure price options (i.e., all miles or all money). If they have linear valuations, they will be indifferent between all of the choices and thus all pricing options would be picked in the same proportion.

The rankings collected fall into 24 distinct response profiles (See Table 3). Only seven of these profiles (profiles $2,4,13,14,17,20$, and 22 ; or 32 respondents) exhibit a corner solution (i.e., respondents prefer to pay in a single currency). The other 17 profiles reveal that the majority of respondents (67 out of 99) prefer a combined-currency price to paying in either all miles or all dollars. This preference for a combined-currency price demonstrates that their
perceived cost curves are not strictly concave for both currencies. Further, if we look at the distribution of first choices ( see last line of Table 3), we have can safely reject ( $p<0.0001$ ) the possibility that these first choices were made at random. Thus, we find support for neither the concave nor the linear valuation hypotheses; hence, there must exist some convexity in most of our respondents' perceived cost functions.

Going one step further, using the linear programming software LINDO (Schrage 1997), we can check if individual respondents rank-ordered all five prices in a way that is consistent with two concave cost functions, including those 32 who picked a single-currency price as their preferred option. We analyze the data by imposing only that each cost function is either concave, convex, or S-shaped without imposing any specific functional form (See Appendix B for a detailed description of the methodology). We specify the restrictions on each of the utility functions independently so that, a priori, one could see any mix of shapes across miles and money. If LINDO could solve the linear program without violating any constraints, then the profile was classified as consistent with a hypothesis of concavity for both miles and dollars (Y in column CC-CC of Table 3). For those favoring an interior solution (i.e., their first choice was a combined-currency price), their orderings clearly violated the concavity restrictions on both miles and dollars. In addition, we find that two profiles (17 and 22, in italics in Table 3) violated the concavity restrictions on both miles and dollars. Hence, 69 out of 99 subjects (70\%) expressed preferences inconsistent with purely concave perceived cost functions.

We should point out that the data are also consistent with a S-shaped cost function for miles, but many more choice options would be required to distinguish between a convex and an S-shaped perceived cost function for miles, which for our purposes is unnecessary. Further, as a testament to respondents adhering to the instructions, and a sign that individual wealth effects do not appear to be driving choice in this case, the average number of miles owned by respondents
did not differ depending on whether they exhibited a corner solution or interior solution ( $p=$ $0.21)$.

## Discussion

In summary, the results from this study reveal a strong preference for an interior solution among experienced fliers, who chose from the identical, richer set of pricing options. Within the range of miles and dollars utilized, we can rule out concave-concave perceived cost functions for those two-thirds of respondents who favored a combined-currency price. In addition, while a concave-concave scenario could apply to 30 of the 32 respondents who favored a corner solution, a concave-convex scenario could apply to the entire sample. Recall from Case 3 of our mathematical proofs that even when the perceived cost function for one currency is concave and the other convex, a corner solution (preference for paying in a single currency) is entirely possible. This is especially true since we test only a discrete number of possible prices combinations rather than the continuous range from 0 to $100 \%$ dollars. It is worth pointing out that while the $50-50$ option was the preferred pricing schedule in Study 2 (it was the only combined-currency price available), the same mix was the least favored option among the broader set of pricing schedules available in Study 3. This highlights the importance of determining the right mix among currencies when offing combined-currency price.

## CONCLUSION, LIMITATIONS AND FUTURE RESEARCH

The objective of this research was to determine the conditions under which what we have labeled a combined-currency price can be superior to a price charged in a single currency. In doing so, we seek to help explain the recent emergence and increasing proliferation of combinedcurrency prices in the marketplace. Our mathematical proofs show how non-linear value functions
result in many instances where marketers should charge combined-currency prices, specifically when the perceived cost function for one of the currencies is convex within the range in question. A combined-currency price in this case could maximize the amount of revenue collected given a set psychological cost, or minimize the psychological cost associated with a given price. Both the anecdotal real-world evidence and our experimental evidence illustrate how people often prefer to pay prices comprised of payments in more than one currency.

More specifically, in Study 1, we offered evidence that both a combined-currency price (i.e., interior solution) and a single currency price (i.e., corner solution) can be superior within the same population of consumers, while simultaneously demonstrating a pattern of choice consistent with a convex perceived cost function for one of the currencies involved. In Study 2, we demonstrated how choices made by actual fliers conform to the general predictions laid out in our proofs and are consistent with the results of Study 1. In Study 3, we collected more detailed individual level data, and provided more direct evidence that people who favor a combinedcurrency price must have convexity in the perceived cost function for at least one of the currencies involved. Taken together, these studies provide convergent evidence supporting the expected requirements for, and benefits from, combined-currency pricing.

The studies are not without their limitations. Both studies 1 and 2 rely on aggregate data, while studies 1 and 3 rely on students as respondents. None of the studies specifies for which currency the consumer's perceived cost function is convex, and we never attempt to document the underlying cause(s) for the shapes of the perceived cost functions. We were careful, however, to (a) insure that the results of Study 1 hold using the responses from only those participants who were frequent flier members, and to (b) qualify students in Study 3 based on their experience accumulating and transacting in miles.

In Study 1, respondents may have favored a single-currency price because as the payment grew larger in one currency, the perceived cost of tracking a transaction in the other currency appeared larger. We recognize that the existence of transaction costs, and their relative magnitude, can affect the relative attractiveness of combined-currency prices. This would occur most often with prices that included extremely small amounts in one of the currencies, and thus a firm may be well served to strategically avoid such highly skewed offerings.

While this research highlights the importance of combined-currency prices and illustrates the advantages to firms that implement them well, it is also limited in the sense that it does not go into detail as to how the firm can derive a precise price for a particular customer or set of customers. While we test the general shape of respondents' perceived cost functions in Study 3, we did not measure these functions with a great degree of precision. Practically speaking, a firm could better ascertain the shape of a particular individual's perceived cost function for specific currencies using a more complete conjoint analysis design. This process would be far more burdensome in terms of the time required by each participant, and hence far more costly than what we have done here. Developing a cost effective way to do this directly for a population or segment of interest would appear to be an important avenue for future work.

Combined-currency prices are designed to minimize the psychological cost associated with a particular revenue objective by taking advantage of people's inability, reluctance, or lack of desire to convert amounts assessed in one currency into denominations of the other currency (Nunes and Park 2003). Of course, important individual differences may moderate the effectiveness of combined currency prices. It may be that certain consumers contemplating spending miles, dollars or any mixture of two currencies may have difficulty recalling or constructing a value for varying amounts of either or both. Or, that for certain consumers, the relevant information is unavailable or inaccessible. This research did not explore the process by
which consumers assess the perceived cost associated with various increments of each currency or compare price schedules. Future research may be directed towards developing a process level model of choice involving prices in combined currencies with an eye towards those factors that mediate or moderate their attractiveness.

The process consumers utilize is likely to be affected by the comparability of the currencies and their knowledge or experience with each. In the same way that Johnson (1984) suggests experts should be more likely to use within-attribute strategies for comparing alternatives, we'd expect those who fly all the time to possess or construct some sort of exchange rate in order to make the relevant comparisons across price schedules. As within-attribute strategies call for comparing attributes directly, we'd expect regular fliers to compare $\$ 39$ plus 16,000 miles versus $\$ 189$ by simply matching dollars to dollars and miles to miles. Hence, they would be left to determine whether paying $\$ 150$ or 16,000 miles is preferable. They might do so by applying their idiosyncratic exchange rate, a well-known exchange rate ( 2 cents), or by considering how long it takes to accumulate each (converting units in each into time). Conversely, non-experts (i.e., infrequent fliers) may utilize what Johnson calls an across-attribute process, making an overall evaluation in which comparisons are made at a very abstract level. This would correspond to these consumers simply asking themselves, "What hurts more, $\$ 189$ or $\$ 39$ plus 16,000 miles?" The latter process seems more in line with the choice patterns we observe, leading us to suspect our results are consistent with the behavior of most consumers who fly relatively infrequently.

With increased exposure and experience, the conversion between two or more particular currencies could, in theory, become second nature. If this were the case, combined-currency prices across these currencies might be expected to lose their efficacy. However, the multitude of buy and sell rates in the marketplace suggests this type of flawless interchangeability is unlikely,
although future research could explore how firms could further obfuscate the value of alternative currencies such as miles or points in terms of other currencies. Perhaps firms are already doing this intentionally through their offerings, but practices that increase or decrease the ease with which consumers compare price schedules alternatives seems ripe for work by researchers interested in pricing.

In this research, we have focused on the perceived cost associated with surrendering various amounts of currencies. The mathematics should apply whether one is surrendering or acquiring the amounts in question. Another interesting avenue for future research may be to investigate any asymmetries in valuation based on whether one expects to give or receive payments in more than one currency. The wife of one author recently inquired as to whether giving a $\$ 100$ gift certificate from a particular restaurant would be perceived as more generous than a $\$ 50$ restaurant certificate and $\$ 50$ in movie passes. She was convinced that one $\$ 100$ certificate would seem grander. Her idea did not go unnoticed and we have begun exploring this question in more detail. In addition, while we have speculated that goals can cause local convexities in the perceived cost functions for currencies, we do so in the absence of wealth effects. It would be interesting to examine not only how expenditures are perceived in relation to goals, but also how asset levels in the relevant currencies can affect decision-making. To this end, exploring how goals and asset levels interact to affect choice seems like a particularly relevant and important area for future research, one that we have already begun studying.

## ENDNOTES

1. This claim was made on its web site in July 2001. At the time the airlines Milepoint.com partnered with included Delta Air Lines, Northwest Airlines, Continental Airlines, US Airways, America West Airlines, Midwest Express Airlines, Hawaiian Airlines, Hilton Hotels, and American Express Membership Rewards. Members could spend their miles at online retailers including Amazon.com and Skymall, as well as such premium retailers as Hammacher Schlemmer and The Sharper Image.
2. Most sources suggest consumers should value miles between one and two cents per mile, which is the exchange rate we use in our experiments. Business Week (1999) reports that, "conventional wisdom says a mile is worth about 2 cents - around what major airlines charge companies that buy mileage for incentive awards to employees or clients." In reality, airlines include a contingent liability in their accounts to cover the cost of unredeemed miles, which are valued at marginal cost - a lot less than what they sell them for (Economist 2002). For example, when a travel award level is attained for a member of United Airlines, the liability is recorded for the incremental costs of providing travel, based on the expected redemptions. The incremental costs include the additional costs of providing service to the award recipient, such as fuel, a meal, personnel and ticketing costs, for what would otherwise be a vacant seat. As of December 31, 1999, the estimated number of outstanding awards was approximately 7 million, of which UAL estimated that only 5.8 million awards would ultimately be redeemed and, accordingly, recorded a liability of $\$ 175$ million. Considering only those miles UAL expected to be redeemed, the airline might be considered as valuing each outstanding mile at about $12 / 100$ ths of one cent. While other miles are redeemed for goods and services at an undisclosed rate, our discussions with marketing and pricing managers from various airlines indicate that mileage promotions involve so few miles, relatively speaking, that a constant or linear valuation from the airline's perspective is an entirely reasonable assumption.
3. We can always set $c_{1}=c_{2}^{\prime}=\alpha c_{2}$.
4. Situation 1 is a special case where although $g$ is S-shaped, its slope at the origin is steeper than the slope of $f$ at that point $\left(f^{\prime}(0) \leq g^{\prime}(0)\right)$. This results in the convex portion of $g$ to be above f . Thus $f^{\prime}(\mathrm{x})$ in the convex portion of the S will always be greater than $g^{\prime}(\mathrm{y})$ regardless of the value of $y$. This yields a result such that it will never pay to split payments between $C_{1}$ and $C_{2}$.
5. An analysis of the data including all respondents gave equivalent results.

TABLE 1

## SOME COMMON REWARDS AND THEIR VALUE IN DOLLARS ON 11/28/2001

| Reward | Provider | Miles | Dollars | Exchange Rate |
| :--- | :--- | :--- | :--- | :--- |
| Palm Pilot VII | American Airlines/AOL Program | 78,500 | $\$ 227^{*}$ | $\$ 0.0029$ |
| Admirals club <br> Membership <br> (add a spouse) | American Airlines | 40,000 | $\$ 300$ | $\$ 0.0075$ |
| $\$ 500$ of closing | Citibank Home Mortgage | 25,000 | $\$ 150$ | $\$ 0.0060$ |
| Time Magazine | Milepoint.com | 25,000 | $\$ 500$ | $\$ 0.02$ |
| Upgrade Award | United Airlines | 900 | $\$ 24.95^{* *}$ | $\$ 0.027$ |

[^0]TABLE 2
Study 1: Percentage Preferring Pure vs. Combined-Currency Prices

| Relatively Low <br> Total Cost | Part 1 <br> \% preferring <br> payment option | Part 2 | Part 3 |
| :--- | :---: | :--- | :--- |
| $\$ 300$ or 30,000 miles |  |  |  |
| $\$ 250+\$ 50$ | $30 \%$ | $75 \%$ |  |
| $\$ 250+5,000$ miles | $\underline{70 \%}(63)^{*}$ |  | $75 \%$ |
| 25,000 miles $+\$ 50$ | $65 \%$ | $25 \%(59)^{*}$ |  |
| $25,000+5,000$ miles | $35 \%(63)^{* *}$ | $25 \%(55)^{*}$ |  |

Relatively High
Total Cost
$\$ 1,050$ or 105,000 miles
$\$ 1,000+\$ 50 \quad 79 \% \quad 18 \%$
$\$ 1,000+5,000$ miles $\quad \underline{21 \%}(63)^{*} \quad 16 \%$
100,000 miles $+\$ 50 \quad 30 \%^{\mathrm{b}} \quad 85 \%(61)^{*}$
$100,000+5,000$ miles $\quad 70 \%(63)^{*} \quad 82 \%(63)^{*}$

Note: Numbers in parentheses indicate the number of responses for each pair. Responses exceed the number of participants as many responded to more than one choice question. Pairs with * differ significantly at $\mathrm{p}<0.01$. Pairs with $* *$ differ significantly at $\mathrm{p}<0.05$.

TABLE 3

## Study 3: Response Profiles



NOTES: * To be read: 25 respondents rank ordered the 5 prices, from best to worse, as $\$ 560 \& 7 \mathrm{~K}, \$ 140 \& 28 \mathrm{k}$, $\$ 700,35 \mathrm{k}, \$ 350 \& 17.5 \mathrm{~K}$. The preferred price bundle is an interior point. For optimal choices, it cannot occur if both currencies exhibit concave utility functions.
** To be read: 8 out of 99 respondents choice $\$ 700 \& 0 \mathrm{~K}$ as their preferred form of payment.

Figure 1: Concave-Concave (Psychological Costs and Optimal Prices)


Figure 2: Convex-Convex (Psychological Costs and Optimal Prices)


Figure 3: Concave-Convex (Psychological Costs and Optimal Prices)


Figure 4: Concave-S-shaped (Psychological Costs and Optimal Prices)


FIGURE 5a
Study 2: Count of choices by ticket prices


FIGURE 5b

Study 2: Probability of Choosing a Particular Pricing Plan as a Function of Price


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## APPENDIX A

The problem faced by the firm is whether to set a price using one or both of two possible currencies will minimize the psychological cost to the consumer while still yielding the desired revenue objective. In mathematical terms, the problem can be defined as follows:

$$
\begin{array}{ll}
\text { Min } E=f\left(c_{1}\right)+g\left(c_{2}\right) \\
\text { st. } & h_{1}: c_{1} \geq 0  \tag{A.1}\\
& h_{2}: c_{2} \geq 0 \\
& h_{3}: c_{1}+c_{2}=r
\end{array}
$$

The first two constraints, $h_{1}$ and $h_{2}$, simply insure prices are non-negative; the third constraint, $h_{3}$, is the revenue constraint.

Since the domain over which we minimize the psychological cost is limited to $[0, r]$ in both $C_{1}$ and $C_{2}$, and both $f$ and $g$ are continuous over this domain, we know by the Weierstrass Theorem (Sundaram 1996) that a minimum exists and we can combine the Lagrange theorem and the Kuhn and Tucker theorem to characterize it. The Lagrangean for the minimization is as follows:

$$
\begin{equation*}
L\left(c_{1}, c_{2}, \lambda_{1}, \lambda_{2}, \lambda_{3}\right)=f\left(c_{1}\right)+g\left(c_{2}\right)-\lambda_{1} c_{1}-\lambda_{2} c_{2}-\lambda_{3}\left(c_{1}+c_{2}-r\right) \tag{A.2}
\end{equation*}
$$

The critical points of this Lagrangean are the solution to the following system of equations:

$$
\begin{align*}
& \frac{\partial L}{\partial c_{1}}=f^{\prime}\left(c_{1}\right)-\lambda_{1}-\lambda_{3}=0  \tag{A.3}\\
& \frac{\partial L}{\partial c_{2}}=g^{\prime}\left(c_{2}\right)-\lambda_{2}-\lambda_{3}=0  \tag{A.4}\\
& \lambda_{1} \geq 0, c_{1} \geq 0, \lambda_{1} c_{1}=0  \tag{A.5}\\
& \lambda_{2} \geq 0, c_{2} \geq 0, \lambda_{2} c_{2}=0  \tag{A.6}\\
& \frac{\partial L}{\partial \lambda_{3}}=c_{1}+c_{2}-r=0 \tag{A.7}
\end{align*}
$$

In this system, constraint $h_{3}$ will always be binding, while constraints $h_{1}$ and $h_{2}$ may or may not be binding. (It is clear that $h_{1}$ and $h_{2}$ cannot be binding simultaneously since this would not yield
any revenue for the firm). Hence, we have three candidates for the minimum, one where only $h_{3}$ is binding, one where both $h_{1}$ and $h_{3}$ are binding, and finally one where both $h_{2}$ and $h_{3}$ are binding.

## Case 1: only $\boldsymbol{h}_{\mathbf{3}}$ is binding - Interior Solution

Since only $h_{3}$ is binding, then we have $c_{1}>0$ and $c_{2}>0$, and thus per equations (A.5) and (A.6), we have $\lambda_{1}=0$ and $\lambda_{2}=0$. This yields:

$$
\begin{align*}
& f^{\prime}\left(c_{1}\right)=\lambda_{3} \\
& g^{\prime}\left(c_{2}\right)=\lambda_{3}  \tag{A.8}\\
& \Rightarrow f^{\prime}\left(c_{1}\right)=g^{\prime}\left(c_{2}\right)
\end{align*}
$$

In other words, a set of prices $\left(c_{1}, c_{2}\right)$ that satisfy the constraint that $c_{1}+c_{2}=r$ such that both $c_{1}$ and $c_{2}$ are strictly positive are local minimums if and only if $f^{\prime}\left(c_{1}\right)=g^{\prime}\left(c_{2}\right)$.

## Case 2: both $\mathbf{h}_{\mathbf{1}}$ and $\mathbf{h}_{\mathbf{3}}$ are binding - Corner Solution

If both $h_{1}$ and $h_{3}$ are binding then we have $\mathrm{c}_{1}=0$, and $\mathrm{c}_{2}=\mathrm{r}$ and per equation (A.6) we also have $\lambda_{2}=0$. This yields:

$$
\begin{align*}
& f^{\prime}(0)-\lambda_{1}=\lambda_{3} \\
& g^{\prime}(r)=\lambda_{3}  \tag{A.9}\\
& \Rightarrow f^{\prime}(0)-\lambda_{1}=g^{\prime}(r) \\
& \Rightarrow f^{\prime}(0)>g^{\prime}(r)
\end{align*}
$$

In other words, for $(0, r)$ to be a possible minimum, it has to be the case that $f^{\prime}(0)>g^{\prime}(r)$.

## Case 3: both $h_{\mathbf{2}}$ and $\mathbf{h}_{\mathbf{3}}$ are binding - Corner Solution

Case 3 is symmetrical to case 2. It yields $c_{1}=r, c_{2}=0$ and $g^{\prime}(0)>f^{\prime}(r)$.

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## APPENDIX B

## Analysis of Study 3 Data

Study 3 can be seen as a conjoint experiment where we have a product (a plane ticket) with two attributes (miles and dollars paid); each attribute has five levels (\$0-\$700 and 0-35,000 miles). If we were to attempt to recover the part-worths associated with each level of each attribute using traditional conjoint method we would require respondents to rank order a minimum of nine profiles.

One can reduce the number of profiles needed by putting constraints on the utility functions. For example, if we knew that the utility for miles $(m)$ and dollars $(d)$ were both linear and the utility associated with 0 miles or 0 dollars were 0 , we could write the following utility function: $U(m, d)=\alpha m+\beta d$. This function has only two parameters $(\alpha, \beta)$ and thus can be easily estimated using our ranking of five products. This type of analysis requires us to know a priori the shape of the utility function with certainty. Because the goal of our exercise is to discover the shape of the utility function, we could not take this approach unless we were prepared to test every possible shape, which is clearly impractical and unfeasible.

An alternative approach is to interpret the rankings strictly for what they are - an indication that the sum of the part-worths of one product is bigger than the sum of the partworths of the other product - and build a linear program that describes the choices made by each respondent. If we look at the preferred choice in our experiment (See Table 3), we find that $\$ 560$ plus 7,000 miles is the preferred alternative, followed by $\$ 140$ plus 28,000 miles, then $\$ 700$, 35,000 miles, and finally $\$ 350$ plus 17,500 miles. What this says is that $\$ 560$ plus 7,000 miles is perceived as less costly than $\$ 140$ plus 28,000 miles. In linear programming terms, and assigning $A_{1}-A_{5}$ to the value of $\$ 0-\$ 700$ and $B_{1}-B_{5}$ to the value of $0-35,000$ miles, we can translate this by $\mathrm{A}_{4}+\mathrm{B}_{2}<\mathrm{A}_{2}+\mathrm{B}_{4}$. We can describe the ranking of the five alternatives as:
$\mathrm{A}_{4}+\mathrm{B}_{2}<\mathrm{A}_{2}+\mathrm{B}_{4} \quad(\$ 560+7,000$ miles preferred to $\$ 140+28,000$ miles $)$
$\mathrm{A}_{2}+\mathrm{B}_{4}<\mathrm{A}_{5}+\mathrm{B}_{1} \quad(\$ 140+28,000$ miles preferred to $\$ 700)$
$\mathrm{A}_{5}+\mathrm{B}_{1}<\mathrm{A}_{1}+\mathrm{B}_{5} \quad(\$ 700+0$ miles preferred to $\$ 0+35,000$ miles $)$
$\mathrm{A}_{1}+\mathrm{B}_{5}<\mathrm{A}_{3}+\mathrm{B}_{3} \quad(\$ 0+35,000$ miles preferred to $\$ 350+17,500$ miles $)$
We can add constraints of monotonicity (more dollars are worth more than less dollars) by imposing that $\mathrm{A}_{1}<\mathrm{A}_{2}<\mathrm{A}_{3}<\mathrm{A}_{4}<\mathrm{A}_{5}$ and $\mathrm{B}_{1}<\mathrm{B}_{2}<\mathrm{B}_{3}<\mathrm{B}_{4}<\mathrm{B}_{5}$. We can further add some scaling constraints by imposing that $\mathrm{A}_{1}=\mathrm{B}_{1}=0$, and $\mathrm{A}_{5}=1$.

Finally, we can add concavity or convexity constraints. To impose that the utility for dollars is concave, we would write the following restrictions:

$$
\begin{aligned}
& 2 \mathrm{~A}_{3}-5 \mathrm{~A}_{2}+3 \mathrm{~A}_{1}<0 \\
& \mathrm{~A}_{4}-2 \mathrm{~A}_{3}+\mathrm{A}_{2}<0 \\
& 3 \mathrm{~A}_{5}-5 \mathrm{~A}_{4}+2 \mathrm{~A}_{3}<0 .
\end{aligned}
$$

The first constraint, for example, comes from the re-arranging of $\frac{A_{3}-A_{1}}{350-0}<\frac{A_{2}-A_{1}}{140-0}$. The other constraints are built in a similar fashion. We can write similar restrictions for miles. To impose convexity instead of concavity all one has to do is change the inequalities from $<0$ to $>0$. We can write similar restrictions to test for S-shaped utilities.

## Testing of the concavity-convexity restrictions

We use the linear programming software LINDO (Schrage 1997) to solve the problem of minimizing the sum of $\mathrm{A}_{1}-\mathrm{A}_{5}$ and $\mathrm{B}_{1}-\mathrm{B}_{5}$ subject to the restrictions defined above. We started by imposing the concavity restriction on both miles and dollars and see if a solution existed. If LINDO could solve the linear program without violating any constraints, then the profile was classified as consistent with a hypothesis of concavity for both miles and dollars ( Y in column CC-CC of Table 3); if LINDO reported that the problem could not be solved because of
constraint violation, then the profile was classified as violating the hypothesis on concavity for both miles and money ( N in column $\mathrm{CC}-\mathrm{CC}$ of Table 3 ). We then repeated the process imposing the constraint of a mix of concavity and convexity (e.g., concave dollars and convex miles). All profiles passed this test. Had any profile failed a mix of concave and convex utility functions it would have be tested using S-shaped utility functions. There is no need to attempt to fit an Sshaped function to systems that are solvable using a mix of concave and convex function, as the S-shape restrictions are less stringent than either the concavity or convexity restrictions.

Using this procedure we find that only five of the 24 distinct profiles are consistent with concavity on both miles and money (See Table 3) while all profiles are consistent with concave dollars and convex miles (and therefore with concave dollars and S-shaped miles). These five profiles account for only 30 of the 99 students.

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[^0]:    *Average price on CNET.com
    **Subscription price for $1 / 2$ year at Time.com and price in miles for 27 issues. ***This is UAL's selling price for four 500-mile upgrades.

